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Monte Carlo simulations to evaluate error propagation in computation of thermal power

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ABSTRACT

Data reconciliation is a commonly used technique for correcting random errors in measurement data in the process industry. The technique uses models describing the mutual relationships of process variables related to available measurements. These models are based on knowledge of process physics. Measurement readings are adjusted so that especially mass and energy balances described by the model match. The technique has proven effective in reducing measurement uncertainties. The paper presents a Monte Carlo study of error propagation in data reconciliation of the turbine section of a VVER 440 nuclear power plant. Uncertainties in model parameters describing turbine dry efficiencies and the quality of steam exiting the steam generators are considered in addition to measurement noise. The impact of these factors on estimated reactor thermal power is evaluated, both individually and as joint impacts. For both the measurement signals and the plant parameters, the resulting effect on the uncertainty of thermal power is lower than the 2% uncertainty for reasonable levels of added noise. These results support the use of data reconciliation for reducing the uncertainty in thermal power.

Key Words: Data reconciliation, thermal power uncertainty determination, Monte-Carlo simulations.

1 INTRODUCTION

Data reconciliation is a commonly used technique for correcting random errors in measurement data in the process industry. The technique uses models describing the mutual relationships of process variables related to available measurements. These models are based on knowledge of process physics. Measurement readings are adjusted so that especially mass and energy balances described by the model match. The technique has proven effective in reducing measurement uncertainties.

Uncertainties are present also in the process model and its model parameters. Some of the uncertain model parameters may be adjusted in data reconciliation that simultaneously minimizes both measurement and model errors. However, usually there is not enough analytic redundancy to correct all uncertainties in the models effectively. Model parameters that are not adjusted in data reconciliation are normally input without an uncertainty and their influence on the calculated results is unknown.

In nuclear power plants, the reactor thermal power is an important safety parameter. The determination of thermal reactor power is traditionally done by establishing the heat balance around the steam generators for PWRs or the reactor core for BWRs. The uncertainty in the reactor thermal power calculation is not always rigorously determined. Instead, the uncertainty is often assumed by regulators to be less than 2% allowing an NPP to operate at 100% power with a safety limit of 102% licensed thermal power [1]. The contribution to the uncertainty in thermal power calculation from different process parameters is difficult to assess analytically. Thus, the use of Monte Carlo simulations as a statistical technique is used to aid this assessment.

The paper presents a Monte Carlo study of error propagation in data reconciliation of the turbine section of a VVER 440 nuclear power plant. Uncertainties in model parameters describing turbine dry efficiencies and the quality of steam exiting the steam generators are considered in addition to measurement noise. The impact of these factors on estimated reactor thermal power is evaluated, both individually and as joint impacts. The results from the study indicate the uncertainty in the studied parameters must be considerable before the uncertainty in the reactor thermal power approaches the 2% safety margin.

With data reconciliation, Gaussian probability distributions are commonly used to represent measurement and model uncertainties. However, the modelled values of turbine dry efficiencies and steam quality values are rather close to their theoretical maximum of 1, which renders Gaussian distribution function unsuitable to represent the corresponding uncertainties. The data reconciliation model deployed in this study has been in use for several years for detection of gross errors. This use has validated it as a reasonable representation of the turbine section.

2 DATA RECONCILIATION

With data-reconciliation [2], [3] data is deduced from the combination of measurements and a physical model. The underlying assumptions are that the measurement errors have known distribution and that the measurement values can be linked by physical equations. The measurement errors are commonly assumed to follow the normal distribution. The physical equations are obtained using a flowsheet-based model.

The object function to be minimized for the data-reconciliation is defined as

$$min\chi^2 = \sum_i \left(\frac{\hat{x}_i - x_i^+}{\sigma_{x_i^+}}\right)^2 \tag{1}$$

subject to

$$g(\hat{x}_i) = 0 \tag{2}$$

where $\hat{x}_i$ are the calculated or reconciled values, x_i^+ the measured values, $\sigma_{x_i^+}$ the measurement standard deviation, and $g(\hat{x}_i)$ are the heat and mass balance equations to be fulfilled. The object function, χ^2 , is used as the primary indication of how well the model fits to the modelled process.

In addition to the global indicator, χ^2, statistical analysis can be used to assess the fit of individual measurements. This results in a value for the standard distribution of the individual measurements deviation to its calculated value:

$$v_i = \hat{x}_i - x_i^+ \tag{3}$$

This deviation is called the residual and its standard deviation σ_v is given by:

$${\sigma_v}^2 = {\sigma_{x^+}}^2 - {\sigma_{\hat{x}}}^2 \tag{4}$$

The ratio of v/σ_v can be used to assess the statistical significance of the deviation. This ratio is, however, only of importance if the analytic redundancy of the measurements is sufficient. That is, if the calculated value is influenced by other measurements or model parameters. We therefore define the term adjustability as

$$a_i = 1 - \sigma_{\hat{x}_i}/\sigma_{x_i^+} \tag{5}$$

The constraint equation, $g(\hat{x}_i) = 0$, holds information about variable parameter values and process states also at un-measured process locations. Thus, data reconciliation gives a set of most probable process states in terms of properties flow, pressure and enthalpy in all parts of the process from which other properties, e.g. temperature, specific volume, entropy and steam quality, can be derived. Also, the standard deviations of the process properties are calculated.

3 THERMAL POWER UNCERTAINTY DETERMINATION

The reactor thermal power is a very important safety parameter and as such the uncertainty in this value is also of importance. The thermal power is normally determined from analysis of a physical model of the power plant. The largest part of the thermal power is determined from a calorimetric calculation of heat exchanged in the boiling part of the process, either in the reactor core for BWRs or in the Steam Generators for PWRs [4].

3.1 Calculation of thermal power by heat balance

The following simplified example shows how to calculate the thermal power of a steam generator by subtracting the heat of the feedwater into the steam generator from the heat of the steam going out of the steam generator, see Figure 1.

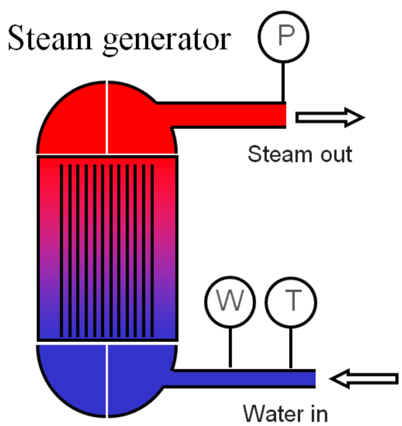

Figure 1. Steam generator with measurement locations.

The measurements used are flow and temperature of the feedwater and steam pressure. In this example we disregard small contributions to the thermal power, e.g. the blow down flow and piping heat losses. The thermal power is then calculated from

$$Q_{SG} = w_{FW}(h_{st} - h_{FW}) = w_{FW} h_{SG} \tag{6}$$

where we for convenience say that $h_{st} = h_g(p)$ and $h_{FW}(T,p) \sim h_f(T)$.

Assuming all quantities are uncorrelated, the uncertainty in steam generator power is then given by

$$\sigma_{Q_{SG}}{}^2 = w_{FW}{}^2 \, h_{SG}{}^2 \left(\frac{\sigma_{w_{FW}}{}^2}{w_{FW}{}^2} + \frac{\sigma_{h_g}{}^2 + \sigma_{h_f}{}^2}{h_{SG}{}^2} \right) \tag{7}$$

The analytic calculation of the uncertainty in steam generator thermal power would be facilitated by further simplifications, e.g. if we further assume that the uncertainties of the dependent quantities i.e. σ_{h_f} and σ_{h_g} are equal to the measurement uncertainties σ_p and σ_T. However, such assumption would not be entirely valid as the enthalpies have a non-linear relation to the pressure and temperature.

3.2 Calculation of thermal power with data reconciliation

Data reconciliation can be utilised for thermal power calculation either by changing the measured values to the reconciled values and their reduced uncertainties in the calculations in Section 3.1 or by using the calculated un-measured process values directly. The method of using data reconciliation to reduce thermal power uncertainty, or measurement uncertainty recapture (MUR) is currently in use in nuclear power plants [4].

When using data reconciliation, the uncertainties in the measurements are reduced. Instead of using only the measurements directly involved in Eqs. (6) and (7), data reconciliation also takes surrounding measurements, process parameters and modelling equation into account. Expansion of the modelled process section allows deployment of more analytic redundancy. Figure 2 shows a schematic process diagram for a steam generator and turbine cycle.

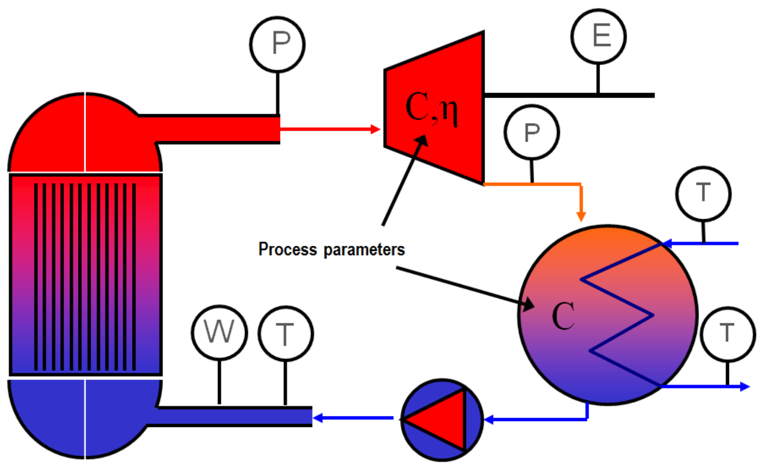

Figure 2. Simplified process diagram with steam generator and turbine cycle components.

The process parameters and measurements are linked through the constraint equation, see Equation (2), and their statistically most likely values are found upon minimising the object function, Equation (1).

The process parameters used in the data reconciliation model are often not assigned an uncertainty and the overall model uncertainty is not fully evaluated. Thus, the contribution of the thermal power uncertainty from the inherent model uncertainty needs to be evaluated.

4 MONTE CARLO SIMULATIONS TO DETERMINE THERMAL POWER UNCERTAINTY

The reliability of the results from data reconciliation depends on the uncertainty of the process model as well as the uncertainties of the process measurements. Evaluation of the uncertainties of non-linear models is best carried out by Monte Carlo simulation, where many experiments are carried out computationally [5]. In this work, the impacts of uncertainties in measurement readings, turbine stage dry efficiencies, and the steam quality of generated steam were evaluated by running tens of thousands of data reconciliations with pseudo random noise added to the relevant variables. Stage dry efficiencies and steam quality are essential parameters in modelling the thermodynamics of steam turbine sections, and hence in estimating the thermal power of a reactor. However, determining their accurate values can be laborious.

Monte Carlo simulations were carried out for several levels of uncertainty, starting from zero (no additional noise) to levels where the standard deviation of the estimated thermal power spanned several megawatts. In the figures below, this uncertainty is depicted by three curves: the mean value of the estimated thermal power and ± 95 % confidence limits as functions of the level of simulated uncertainty. Similarly, the values of the object function, Equation (1), are shown as the mean value and ± 95 % confidence limits. The 95% limits are better suited to describe uncertainties in non-linear processes and process models than standard deviations, as the normality of probability distributions cannot be counted on.

Pseudo random numbers were added to all relevant variables in the process model: 287 process measurements, 21 turbine stage dry efficiency parameters, and steam qualities at six steam generators. For slightly more realistic setup, a small level of measurement noise was included in the simulations assessing the impacts of stage dry efficiency and steam quality uncertainties.

For this study, a process model of the turbine section of a VVER-440 reactor developed using the thermal performance monitoring system TEMPO [6] was used. The process contains 6 steam generators and the turbine cycle is divided in two almost identical turbine trains, named the 10-side and 50-side, respectively, each of which is connected to a separate generator. Both sides contain a high-pressure turbine and two low pressure turbines connected to two condensers. Figure 3 shows part of the process model developed in TEMPO. The model includes 264 variable process parameters, 437 components and 500 flows. The number of redundancies in the measurement set is 109.

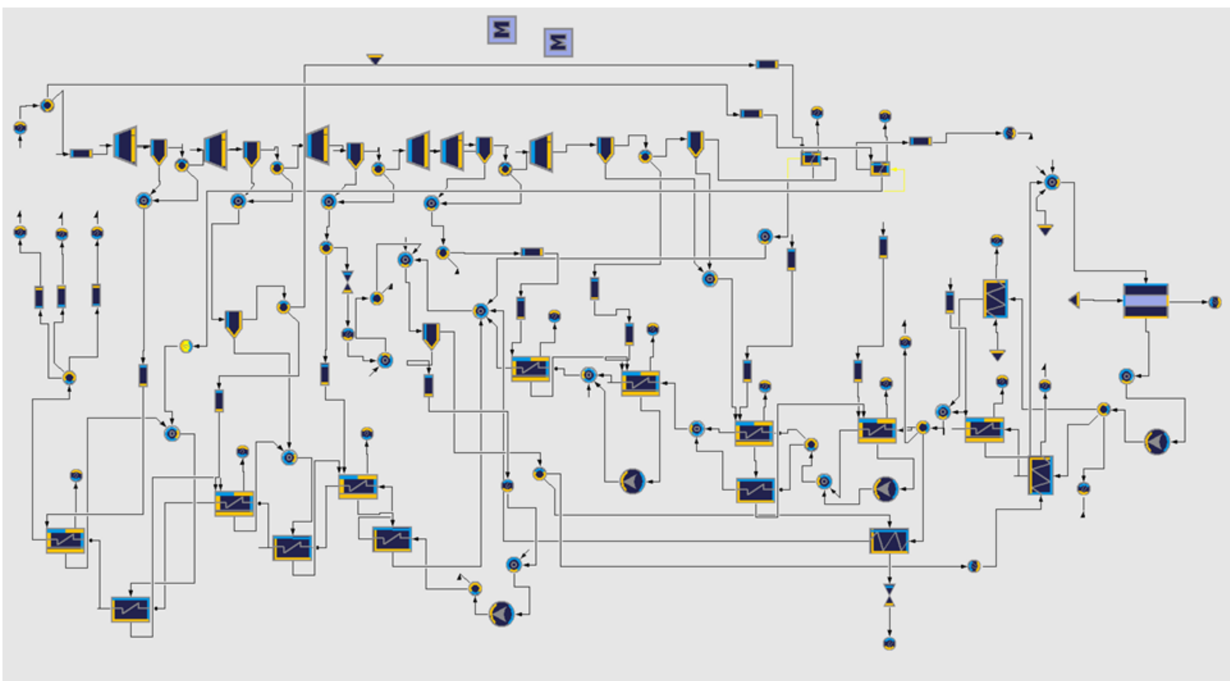

Figure 3. The high-pressure and main condensate part of the 50-side turbine section modelled in the TEMPO system.

The results in the sections below are shown only for the 50-side. The uncertainties on the 10-side correspond to them.

4.1 Uncertainty of measurement readings

The most obvious, and correspondingly most often accounted for source of uncertainty is uncertainty in measurement data. Measurement uncertainties are included explicitly in process models built for data reconciliation. In this study, the modelled measurement uncertainties were utilized. Various levels of measurement uncertainty were simulated by using a coefficient in the range from zero to two to multiply the modelled measurement uncertainty given as either absolute or relative standard deviation. A coefficient value of one indicates measurement uncertainties as modelled. Gaussian distributions are used for the pseudo random numbers.

The modelled uncertainties are often inflated. This is due to available modelling elements (computational modules) being simplified representations of the reality, and available measurements not necessarily being optimally placed regarding to the modelling elements available. Hence, there tends to be a bias between available process measurements and the assumptions made when building the model. To expedite process modelling, this bias is usually included in the modelled measurement uncertainty, as illustrated in Figure 4. This is a reasonable practice in general, but for Monte Carlo simulations like this study it tends to exaggerate the resulting uncertainty.

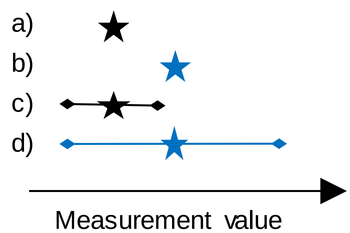

Figure 4. Inflated measurement uncertainty in data reconciliation: a) available reading from process measurement, b) reading assumed in modelling the process, c) uncertainty of the process measurement, and d) measurement uncertainty in the process model is extended to include both bias and uncertainty of the process measurement.

Figure 5 and Figure 6 show the object function and the estimated thermal power at various levels of measurement uncertainty, respectively. The value of the object function increases significantly as the function of measurement uncertainty, but the average of the estimated thermal power remains almost constant as data reconciliation adjusts deviating measurement readings to match mass and energy balances. The 95 % confidence limits extend to ±2 % of the reactor power when the standard deviations indicating measurement uncertainties are approximately double the modelled values

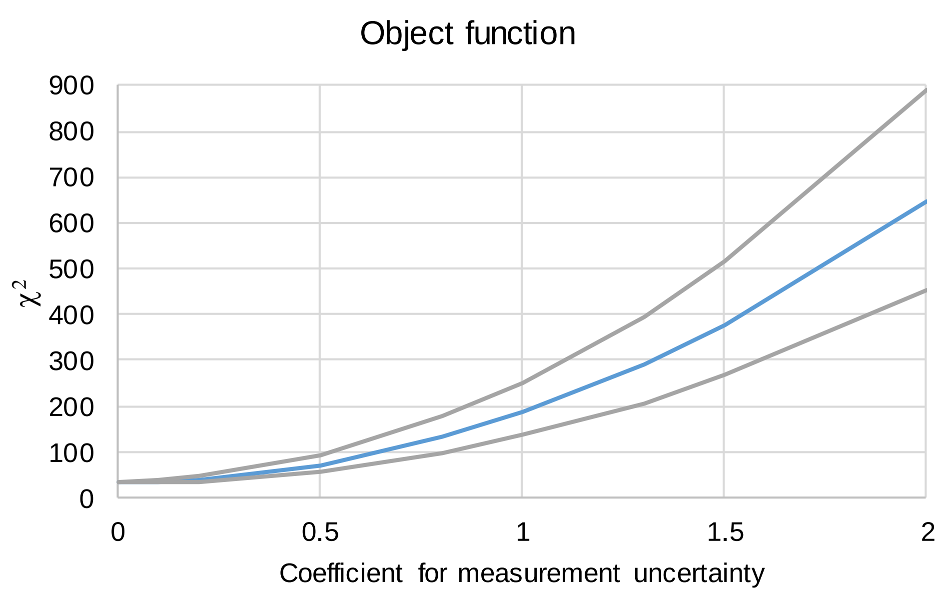

Figure 5. Object function and its uncertainty at different levels of simulated measurement uncertainty. The middle (blue) line shows the average value and the outer (grey) lines its 95% confidence limits.

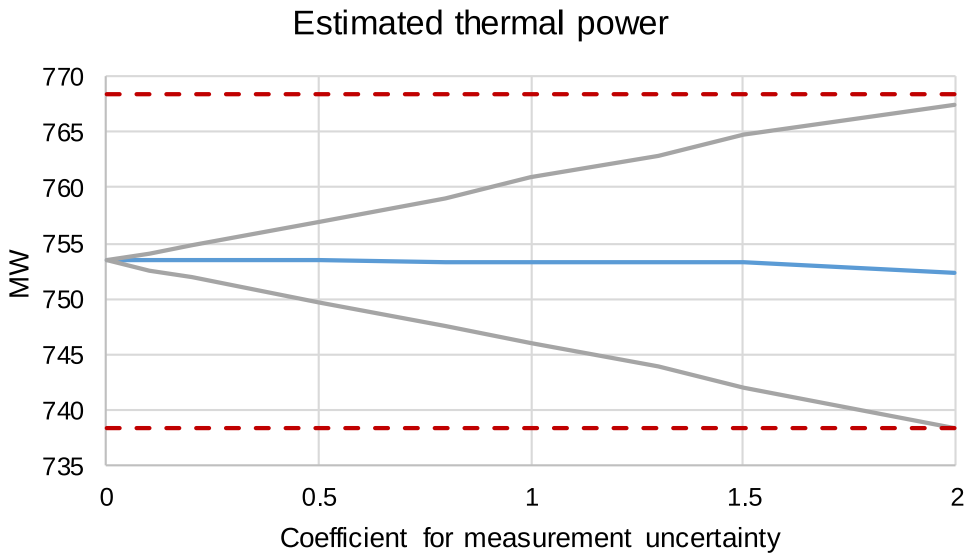

Figure 6. Estimated thermal power (50-side) and its uncertainty at different levels of simulated measurement uncertainty. The dashed horizontal red lines represent ±2 % of the actual thermal power.

4.2 Uncertainty of turbine stage efficiency parameters

The process model of the turbine section has altogether 21 parameters for turbine stage dry efficiencies, including high-pressure and low-pressure turbine stages for both 10- and 50-side. The number of parameters is smaller than the number of the stages (and odd), because some of the stages are defined to have mutually equal dry efficiency values. In Monte Carlo simulation, perturbations to these parameters were introduced with normally distributed pseudo random numbers with standard deviations relative to the original modelled values.

Figure 7 and Figure 8 show the object function and the estimated thermal power at various levels of uncertainty of the stage efficiency parameters, respectively. Graphs are shown only up to turbine uncertainty 7%, before the uncertainty of the reactor power reaches ±2 % of the actual reactor power. This is because at around that point data reconciliation became less likely to converge, see Table I.

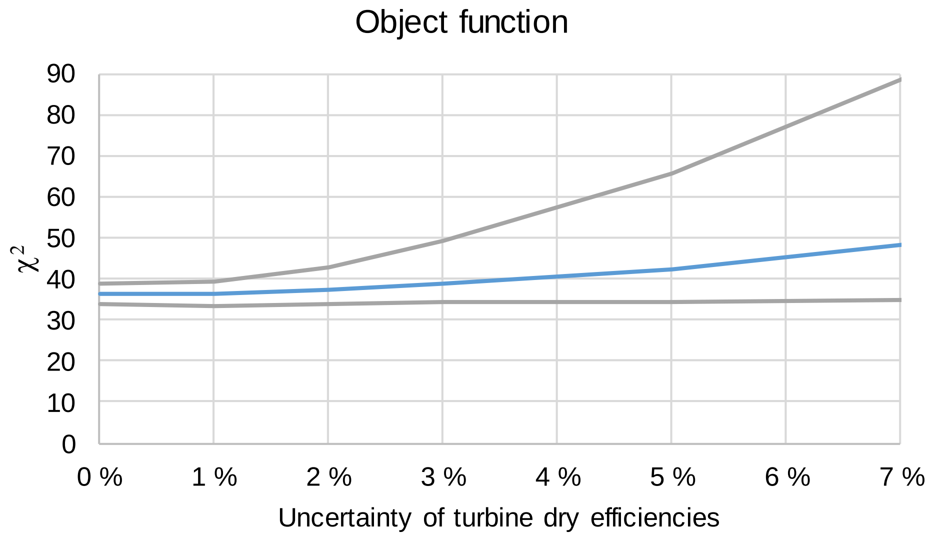

Figure 7. Object function and its uncertainty at different levels of simulated uncertainty of turbine stage dry efficiencies.

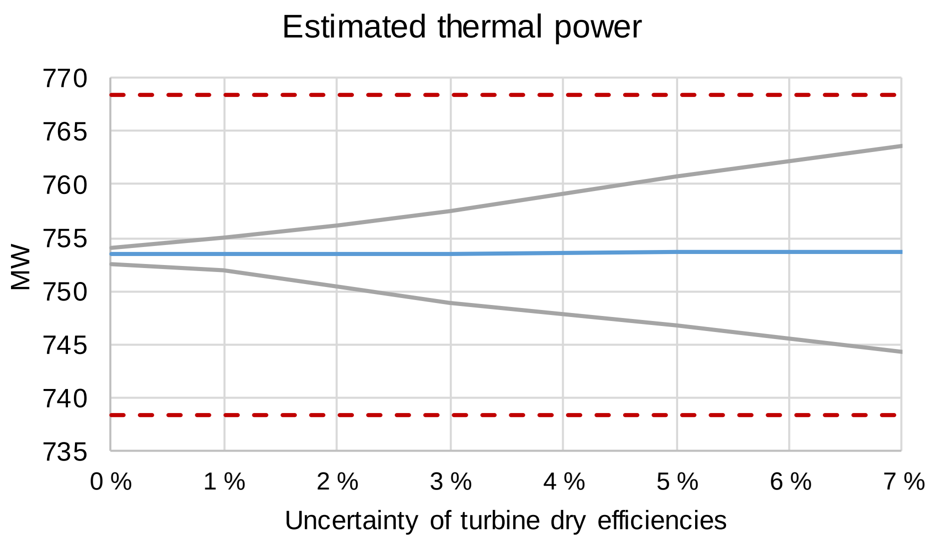

Figure 8. Estimated thermal power (50-side) and its uncertainty at different levels of simulated uncertainty of turbine stage dry efficiencies.

Table I. Convergence rate of data reconciliation at different levels of simulated uncertainty of turbine stage dry efficiencies.

Uncertainty of stage dry efficiencies	Convergence rate of data reconciliation
0 %	100 %
1 %	100 %
2 %	100 %
3 %	100 %
5 %	96 %
7 %	86 %
10 %	70 %
15 %	50 %
20 %	35 %

4.3 Uncertainty of steam quality

The modelled turbine section has six steam generators. The steam quality, i.e. the mass fraction of steam in steam/water mixture, has been modelled as 0.998 for each of them.

The uncertainty of steam quality was simulated using uniformly distributed pseudo random numbers, unlike the experiments in the previous sections where Gaussian distributions were used. Uniform distribution was selected because the original modelled steam quality value, 0.998, is quite close to the theoretical maximum, one, and because the uncertainty in steam quality is quite small. For the smallest simulated uncertainties, the intervals of pseudo random numbers were centred on the original modelled value; for larger uncertainties the maxima of the intervals were set to value one as depicted in Figure 9. The horizontal axes in the figures of this section show the absolute width of the uniform distribution.

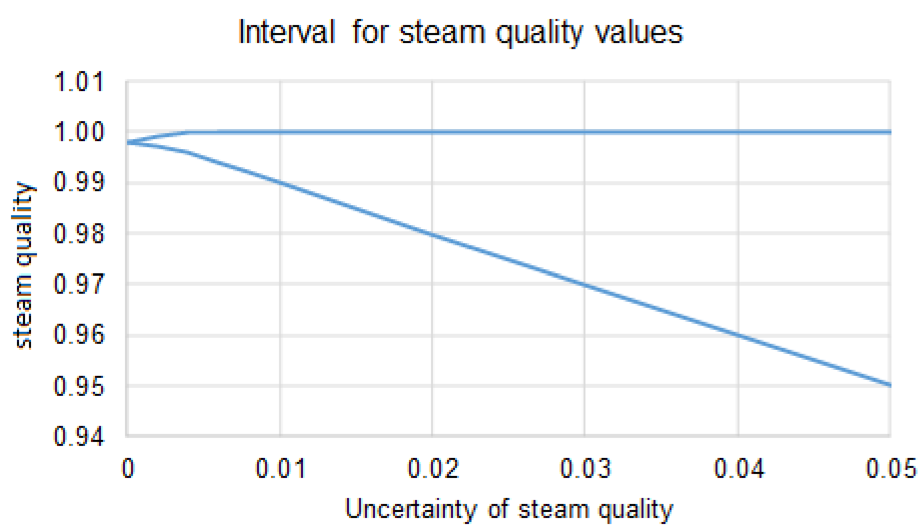

Figure 9. Value ranges of steam quality during simulation. The horizontal axis shows the width of the interval for uniformly distributed pseudo random numbers.

Figure 10 and Figure 11 show the object function and the estimated thermal power at various levels of uncertainty of steam quality, respectively. The steam quality parameters must deviate quite significantly from the original modelled value before the 95 % confidence limit reaches the 2 % limit of reactor power. Due to the closeness to the theoretical maximum and the corresponding shape of the probability distribution in the simulation, the estimated thermal power and its confidence limits are not symmetrical around the actual value.

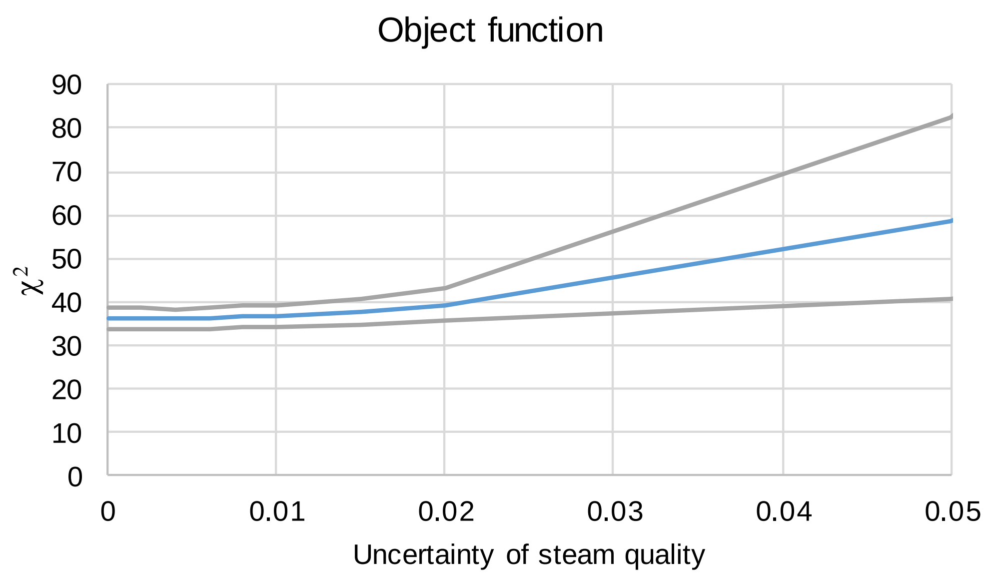

Figure 10. Object function and its uncertainty at different levels of simulated uncertainty of steam quality.

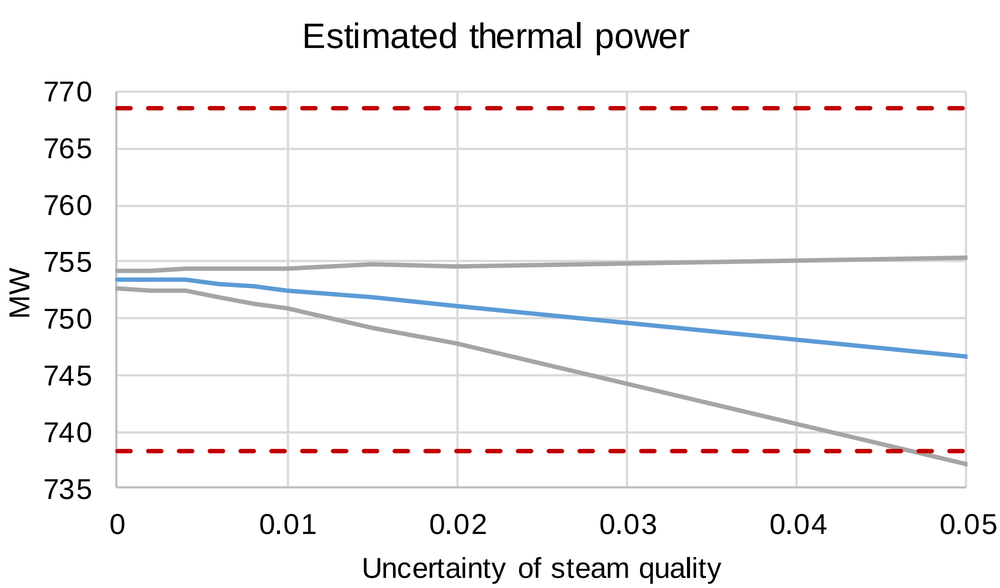

Figure 11. Estimated thermal power (50-side) and its uncertainty at different levels of simulated uncertainty of steam quality.

4.4 Combined uncertainties

The pseudo random numbers representing uncertainties in the Monte Carlo simulation were uncorrelated. If the process model were linear, the impacts of the uncertainties could be combined by adding the variances due to each uncertainty factor separately,

$$\sigma_{Q_{SG}} = \sqrt{{\sigma_{Q_{SG,1}}}^2 + {\sigma_{Q_{SG,2}}}^2 + {\sigma_{Q_{SG,3}}}^2} \tag{8}$$

where the additive terms represent uncertainties due to uncertainties of measurement readings, turbine stage dry efficiencies, and steam quality. To evaluate the joint impact in the non-linear process model, Monte Carlo simulations were executed with several combinations of uncertainties discussed in previous sections. Figure 12 depicts the standard deviation of the simulated thermal power vs. the standard deviation computed with Equation (8). Each point in the figure represent at least 200 simulation runs with a specific combination of levels of uncertainty for measurements, stage dry efficiencies, and steam quality shown individually in the previous chapters. Despite the smaller number of simulations, and hence larger variability than in the previous sections, Equation (8) gives a reasonably good indication of the joint effect.

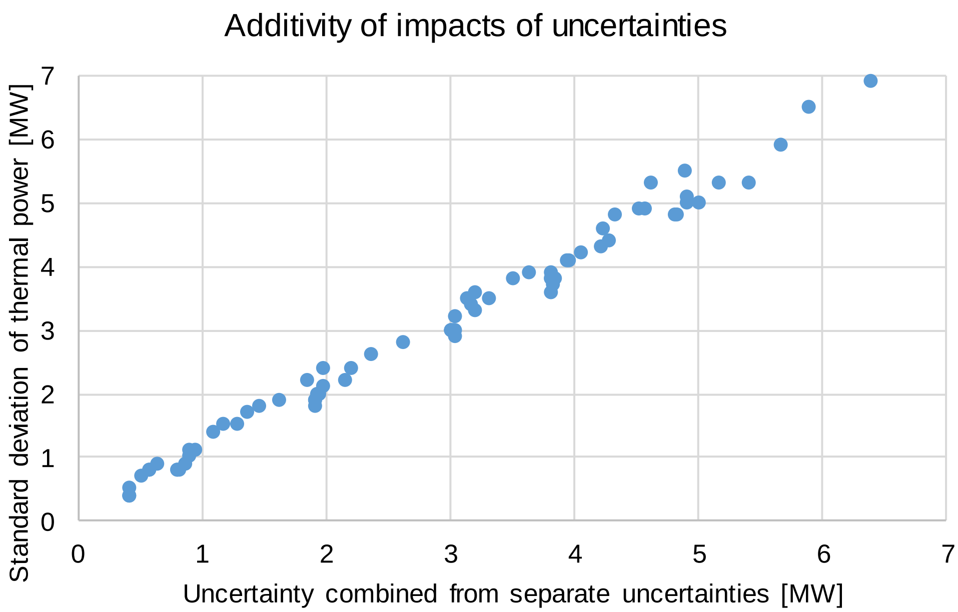

Figure 12. Additive combination of impacts of individual sources of uncertainty (horizontal axis) gives a reasonably good estimate of their simultaneous impact (vertical axis) despite model non-linearities.

5 CONCLUSIONS

The uncertainty of thermal power is an important safety factor. With data reconciliation the uncertainty may be significantly decreased compared to using measurement readings directly. The reliability of the data reconciliation is also dependent on the uncertainty in process parameters as well as the model itself. Monte Carlo simulation was used to assess the influence of process parameters on the thermal power uncertainty. Two plant parameters, turbine stage efficiency and steam generator steam quality, and the measurement signals were simulated with different levels of added noise. For both the measurement signals and the plant parameters the resulting effect on the uncertainty of thermal power is lower than the 2% uncertainty for reasonable levels of added noise. For the plant parameters this implies that the modelling uncertainty when using fixed parameter values is not significantly adding to the thermal power uncertainty. For measurement values, including the measurement bias in the measurement uncertainty as is commonly done when using data reconciliation is also not significantly adding to the uncertainty in calculated thermal power. These results support the use of data reconciliation for reducing the uncertainty in thermal power.

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